

TEMA 2: Ecuación Diferencial Lineal de coeficientes constantes.

$$\text{EDO}(n) \text{ L } \underline{\text{CC}} \begin{cases} \text{H.} \\ \text{NH.} \end{cases}$$

$$a_0 \frac{d^n y}{dx^n} + a_1 \frac{d^{n-1} y}{dx^{n-1}} + \dots + a_{n-1} \frac{dy}{dx} + a_n y = Q(x)$$

Regla
de
Om

"El coeficiente (constante) de la derivada de mayor orden deberá ser la unidad para aplicar todos los métodos de solución"

$$\left| \frac{d^n y}{dx^n} + \frac{a_1}{a_0} \frac{d^{n-1} y}{dx^{n-1}} + \dots + \frac{a_{n-1}}{a_0} \frac{dy}{dx} + \frac{a_n}{a_0} y = \frac{Q(x)}{a_0} \right|$$

$$\text{EDO}(n) \subset \text{ccH}$$

$$\frac{dy}{dx} + ay = 0$$

$$\frac{dy}{dx} = -ay$$

$$dy = -ay dx$$

$$\frac{dy}{y} = -a dx$$

$$\int \frac{dy}{y} = -a \int dx$$

$$\ln y + C_1 = -ax + C_2$$

$$\ln y = -ax + C_2 - C_1$$

$$y = e^{(-ax + C_2 - C_1)}$$

$$y = e^{(C_2 - C_1)} e^{-ax}$$

$$y = C_{10} e^{-ax} \quad \text{SG}$$

$$\frac{dy}{dx} + ay = 0$$

EDO(1) $\subset$ ccH.

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0$$

H₀

$$y = e^{mx}$$

$$\frac{dy}{dx} = m e^{mx}$$

$$\frac{d^2 y}{dx^2} = m^2 e^{mx}$$

Ecuación
Característica

$$m^2 e^{mx} + a_1 m e^{mx} + a_2 e^{mx} = 0$$

$$(m^2 + a_1 m + a_2) e^{mx} = 0$$

$$e^{mx} = 0 \quad mx \rightarrow -\infty$$



$$m^2 + a_1 m + a_2 = 0 \quad \begin{cases} m_1 \\ m_2 \end{cases}$$

Caso I: $m_1 \neq m_2 \in \mathbb{R}$

Caso II: $m_1 = m_2 \in \mathbb{R}$

Caso III: $m_1 \neq m_2 \in \mathbb{C}$

$$\frac{d^2 y}{dx^2} + a_1 \frac{dy}{dx} + a_2 y = 0 \quad \text{EDO}(2) \perp \text{CC} \perp \text{I}.$$

$$m^2 + a_1 m + a_2 = 0 \quad \left\{ \begin{matrix} m_1 \\ m_2 \end{matrix} \right. \quad m_1 \neq m_2 \in \mathbb{R}$$

$$\begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} \neq 0$$

$$y_{\text{gen}} = c_1 e^{m_1 x} + c_2 e^{m_2 x}$$

$$m_2 e^{m_2 x} e^{m_1 x} - m_1 e^{m_1 x} e^{m_2 x} \neq 0$$

$$(m_2 - m_1) e^{m_1 x} e^{m_2 x} \neq 0$$

$$e^{m_1 x} \neq 0 \quad e^{m_2 x} \neq 0$$

$$m_2 - m_1 \neq 0 \quad m_1 \neq m_2$$

$$\left| \frac{d^2 y}{dx^2} - 7 \frac{dy}{dx} + 12y = 0 \right.$$

$$m^2 - 7m + 12 = 0$$

$$(m - 3)(m - 4) = 0 \quad m_1 \neq m_2$$

$$y = e^{3x} \quad y = e^{4x}$$

$$\left| y_g = c_1 e^{3x} + c_2 e^{4x} \right.$$

CASO III.- $m_1 \neq m_2 \in \mathbb{C}$

$$m_1 = a + bi \quad m_2 = a - bi$$

$$y_1 = e^{(a+bi)x} \quad y_2 = e^{(a-bi)x}$$

$$y = C_1 e^{(a+bi)x} + C_2 e^{(a-bi)x}$$

$$y = e^{ax} \left(C_1 e^{(bx)i} + C_2 e^{(-bx)i} \right)$$

$$x \in \mathbb{R} \quad y \in \mathbb{R}$$

$$e^{bi} = \cos(b) + i \operatorname{sen}(b)$$

$$e^{bxi} = \cos(bx) + i \operatorname{sen}(bx)$$

$$e^{-bxi} = \cos(bx) - i \operatorname{sen}(bx)$$

$$y = e^{ax} \left(C_1 (\cos(bx) + i \operatorname{sen}(bx)) + \right.$$

$$\left. + C_2 (\cos(bx) - i \operatorname{sen}(bx)) \right)$$

$$y = e^{ax} \left([C_1 + C_2] \cos(bx) + [C_1 i - C_2 i] \operatorname{sen}(bx) \right)$$

$$y = e^{ax} \left(C_{10} \cos(bx) + C_{20} \operatorname{sen}(bx) \right)$$

$$y = C_{10} e^{ax} \cos(bx) + C_{20} e^{ax} \operatorname{sen}(bx)$$

$$\begin{vmatrix} e^{ax} \cos(bx) & e^{ax} \operatorname{sen}(bx) \\ \frac{d}{dx} (e^{ax} \cos(bx)) & \frac{d}{dx} (e^{ax} \operatorname{sen}(bx)) \end{vmatrix} \neq 0$$

CASO II - $m_1 = m_2 \in \mathbb{R}$

$$\begin{vmatrix} e^{m_1 x} & e^{m_2 x} \\ m_1 e^{m_1 x} & m_2 e^{m_2 x} \end{vmatrix} = 0$$

$$m_1 = m_2$$

$$\begin{array}{ccc} \frac{d}{dm} \left(\begin{array}{c} e^{mx} \\ e^{mx} \end{array} \right) & \xrightarrow{\quad} & e^{m_1 x} \\ & \searrow & \\ & x e^{mx} & \xrightarrow{\quad} x e^{m_1 x} \end{array}$$

